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Beyond normality: Capital market Value-at-Risk modelling using symmetric and asymmetric Laplace distributions

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Abstract

Evidence on parametric VaR employing both Laplace (exhibiting excessive kurtosis) and asymmetric Laplace (additionally being skewed), "tent-shaped" probability distributions is available, although it remains relatively limited. The research procedure in the paper pursued two primary objectives: to back-test both distributions and assess their ability to capture extreme events, and to determine whether the distribution that best fits the entire empirical distribution is also the one that performs best in back-testing in long-term. The indices considered include WIG30, BVP, CAC, DAX, FTM, HSI, NKX, SHC), SPX and TWSE. The tests were performed using daily data for period between 31 December 1998, and 30 June 2025. Across the ten markets studied, both distributions generally outperformed historical simulation and the normal probability distribution, with the asymmetrical Laplace distribution particularly outperforming the Laplace distribution in capital markets that are likely to be skewed for higher confidence levels (0.99 and 0.975) considered.

Keywords

- extreme market risk
- Value-at-Risk
- capital markets

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Introduction

Financial models frequently assume that an instrument's returns are normally distributed; however, this assumption usually deviates from real-world behaviour. Empirical evidence demonstrates that returns commonly exhibit both skewness and excess kurtosis (Altun et al., 2018; C. F. Lee & Su, 2012). High-frequency returns tend to stray from normality, showing heavier tails and more pronounced peaks (Bormetti et al., 2007). These characteristics lead to higher probabilities of extreme, rare events, as well as to outcomes clustered near zero. Additionally, returns often display volatility clustering and asymmetry (Huang & Lin, 2004), and their elevated peaks may be related to low transaction volumes. Acknowledging that returns are non-normally distributed, fat-tailed and skewed is essential for generating accurate forecasts (Braione & Scholtes, 2016). Excess kurtosis may simply result from incomplete data or instruments with relatively low liquidity and a significant number of zero returns.

Capital markets exhibit extreme events that the normal distribution struggles to capture. There are more realistic probability distributions (closer to empirical distributions, exhibiting skewness and excessive kurtosis) than the normal distribution, but they are more difficult to handle mathematically (Wilmott, 2006, pp. 297–299). From the perspective of computational finance, many open-source tools exist that allow financial analysts to easily handle sophisticated theoretical distributions for risk-related actions. Considering that analysts now operate in an era of copilots and AI agents, deriving VaR scripts in popular computer languages is more widely available than ever before. These theoretical distributions, which better fit empirical changes of assets exposed to market risk, include significantly leptokurtic, tent-shaped Laplace distributions.

The existing research provides rather limited empirical evidence on the use of Laplace distributions in parametric financial risk modelling. Prior studies offer selective findings, focusing either on the Laplace distribution for selected individual stocks (Ullah et al., 2022), on the asymmetrical Laplace distribution in the context of currency (Jing et al., 2022), cryptocurrency markets (Subramoney et al., 2021), or agricultural commodities (Živkov, 2025). While papers on indices behaviour also exist (Chen et al., 2024; Taylor, 2019), they focus on limited indices sets and use semiparametric approach. The main novel contribution to the international literature is conducting comprehensive, long-term Value-at-Risk back-tests (additionally involving distribution fitting) for both the symmetrical and asymmetrical Laplace distributions across a broad set of indices representing major capital markets, including WIG30 (Warsaw), BVP (BOVESPA, Sao Paulo), CAC (CAC40, Paris), DAX (DAX40, Frankfurt), FTM (FTSE250, London), HSI (Hang Seng, Hong Kong), NKX (Nikkei225, Tokio), SHC (Shanghai), SPX (S&P500, New York) and TWSE (TAIEX, Taiwan), covering all modern market disruptions (the internet bubble, the financial crisis, the Eurozone crisis, Brexit, COVID-19 and war in Ukraine). A diversified set of equity indices from Europe, the US, Latin America, and Asia was selected to test the VaR model across different market environments. The sample includes both developed and emerging markets.

The research procedure of this paper was designed to achieve two primary objectives: (1) to back-test symmetrical and asymmetrical Laplace distributions and assess their ability to capture extreme events, and (2) to determine whether the distribution that best fits the entire empirical distribution is also the one that performs best in back-testing in long-term. A standard Kupiec test was performed using daily data obtained from stooq.pl for the period between 31 December 1998, and 30 June 2025 (6,379 daily simple rates of return). The safety level, often referred to as VaR in research, was used as the extreme risk measure. A 250-day window was applied.

The findings support earlier evidence that heavy-tailed models generally provide a more reliable measure of extreme market risk than historical simulation and normal models. Across the ten markets studied, the Laplace distribution and its asymmetrical counterpart generally outperformed historical simulation and the normal probability distribution, with the asymmetrical Laplace distribution particularly outperforming the Laplace distribution in capital markets that are likely to be skewed for higher confidence levels (0.99 and 0.975).

The structure of the paper is as follows: Section 1 reviews the literature on VaR models. Section 2 describes and explains the methodology and data employed. Section 3 presents the empirical results, while Section 4 offers a discussion of these findings. The final section summarises the empirical analysis.

1. Literature review

The empirical probability distribution effectively captures historical extreme events until another ‘black swan’ appears. Markets have become increasingly volatile and another unprecedented disruption always lies ahead (e.g., COVID-19, conflict in Ukraine). Using the empirical probability distribution is referred to as either the non-parametric approach or the historical simulation method. To address the limitations of the former HS method an upgrade called the hybrid approach was developed.

The hybrid approach to VaR combines the best of both worlds: historical simulation (HS, non-parametric approach) and parametric approach (Richardson et al., 1997; Zumbach, 2007). Applying HS for VaR avoids the need to choose the most appropriate theoretical distribution: fat tails, skewness, and other specific properties are accounted for directly. The two disadvantages of HS are: 1) problems with calculating quantiles on small samples; and 2) the assumption that returns are identically distributed, which excludes time-varying volatility. The hybrid mechanism instead of assuming equal weights for returns (as HS does) attributes exponentially declining weights to returns and may significantly outperform HS in the consequence (Richardson et al., 1997).

Using an appropriate theoretical distribution could theoretically cover the extreme events and could be considered a clear approach. The question is which theoretical distribution addresses the problem of leptokurtosis and fat tails. The normal distribution assumption might be considered here as an adequate solution when simplicity is the key focus (Wilmott, 2006, pp. 297–299). However, while assets that closely follow the normal distribution actually exist, this phenomenon is uncommon. Setting simplicity aside, there are many theoretical distributions that fit better to empirical distributions or/and reflect extreme risk in a more precise way, as has been proven in many papers.

The empirical distributions are either leptokurtic with fat tails or skewed, and both characteristics often occur simultaneously, a fact that seems crucial. The Student's-T distribution, when applied in APARCH model, has been proven to work better than normal probability distribution for higher confidence levels when compared to RiskMetrics or APARCH using normal probability assumption (Huang & Lin, 2004). Involving standard back-testing procedures, it has been demonstrated that using skewed counterparts of the normal, Student's-T, and Multivariate Exponential Power distributions is important and that the skew-T (SST) outperformed others (Braione & Scholtes, 2016). The skew-generalised-T (SGT) distribution, an extension of skew-T distribution, has been shown to be a superior fit to the empirical distributions of stock indices' log returns compared to normal and Student's-T distributions. Consequently, the SGT assumption resulted in more conservative VaR forecasts. The SGT-based VaR model allows for flexible treatment of skewness, leptokurtosis, and fat tails, unlike the symmetrical Student's-T, which results in better accuracy (C. F. Lee & Su, 2012). The generalised alpha-skew-T (GAST) distribution, also an extension of skew-T, brings uni- or bimodal, skewed and fat-tailed shapes and yields more conservative VaR estimates than the normal, Student's-T, and skew-T distributions (Altun et al., 2018). However, the SGT may be less robust than the α-stable distribution in case of oil price changes (Serrano Bautista & Núñez Mora, 2019).

The Gaussian Mixture Model (GMM) can quickly and adequately adjust to significant and rapid stock market changes when compared with the classic parametric approach, Monte Carlo (both based on the normal distribution assumption), and historical simulation (Morkūnaitė et al., 2024). Research concerning Power GARCH processes assuming stable Paretian or Student's-T distribution indicated that an asymmetrical stable Paretian distributional assumption may be a better solution in the case of currency risk (Mittnik & Paolella, 2003).

An interesting solution for VaR estimation might be the skewed generalised error distribution (SGE). Evidence suggests that GARCH-SGE models may provide more accurate VaR estimates for both low and high confidence levels than GARCH-N models, which proves that incorporating skewness and kurtosis of returns is critical (M. C. Lee et al., 2008). Another evidence confirms that SGE may fit better to the empirical distribution and introduces SGE for option pricing (Theodossiou, 2015).

The generalised hyperbolic distribution (GH) subclasses can be also considered. Kolmogorov-Smirnoff statistic (K-S) and Akaike Information Criterion (AIC) were applied to choose the optimal GH subclass for USD/ZAR daily changes. The variance-gamma subclass proved to be the best choice compared to the classic gaussian assumption (Kemda et al., 2015). However, other evidence indicates that GH performed worse than the SST when applied to S&P500 daily returns (Guo, 2017).

Both the logistic and hyperbolic secant distributions are leptokurtic and fat-tailed. Evidence suggests that these probability distributions provide a much better fit than their popular alternatives when modelling both unconditional and conditional time series distributions (Bagnato et al., 2015). The "tent-shaped" Laplace distribution (L) is symmetrical, significantly fat-tailed, and should result in conservative VaR estimates. Considering the existing research, the Laplace distribution is applicable for VaR and CVaR through GARCH models when compared to the normal and Gumbel distributions (Ullah et al., 2022). Another study provided evidence on the use of generalised autoregressive score (GAS) combined with heavy-tailed generalised lambda distribution (GAS-GL) against GAS models combined with the asymmetrical Laplace distribution (AL), asymmetrical Student's t-distribution (AST), and the skew-T distribution (GAS-SST). GAS-GL occurred to be the most suitable for Bitcoin, while GAS-GL, GAS-SST, and GAS-AL were most effective for Ethereum across various VaR levels (Subramoney et al., 2021). There is also evidence on a semiparametric approach based on asymmetrical Laplace in terms of incorporating overnight returns (Chen et al., 2024).

Tempered stable distributions could be considered as a practical evolution of stable distributions. Evidence exists supporting the application of the classical tempered stable (CTS), normal tempered stable (NTS), and rapidly decreasing tempered stable (RDTS) distributions (Bianchi, 2014). This paper focuses on funds and confirms that the CTS model appears more satisfactory for the daily interval than the normal and other tempered stable models. There is also evidence which refers to multivariate cases. A model using the multivariate normal tempered stable (MNTS) distribution may provide more realistic market risk estimates than the normal distribution assumption (Kim et al., 2012).

Within the framework of Extreme Value Theory (EVT), both Gumbel and Fréchet distributions can be considered useful for covering extreme events, as supported by evidence from South-east and East Asian countries (Carvalhal & Mendes, 2003). These two distributions have asymmetrical shape, with higher probability of extremely low values and lower probability of extremely high values, which may be suitable for certain assets. Another study shows that the generalised Pareto distribution may be applied in expected shortfall (ES) estimation for indices having leptokurtic and asymmetrical distributions during financial crises (Kourouma et al., 2010). Bali (2003) also suggests that "thin-tailed Gumbel and exponential distributions with rapidly decreasing tails are strongly rejected against the fat-tailed Frechet and Pareto distributions with slowly decreasing tails." Further evidence compares models based on EVT, Stable Paretian distributions (both symmetrical and skewed), the normal probability model, and the historical simulation (HS). In the context of VaR, these results showed that fat-tailed models (EVT, Paretian) could predict risk more accurately (Harmantzis et al., 2005). Evidence also suggests that a combination of the normal and Rayleigh probability distributions can be applied to better capture extreme events (Ahmed et al., 2021).

The existing research spans different markets and time periods, supporting the premise that the most suitable model will vary depending on the market, asset, time period, and data interval. According to studies on the Student's-T distribution, the ARCH family of models, and EVT, all of these approaches offer both advantages and limitations. Consequently, no single VaR model can be considered definitive, although all have proven more capable of capturing fat tails than the normal distribution (Lechner et al., 2010).

Whether a theoretical distribution provides a better fit can be assessed using goodness-of-fit statistics, information criteria, or graphical tools such as quantile–quantile plots, probability–probability plots, or density plots. It is even recommended to combine both graphical and non-graphical methods (Vose, 2008), which, when applied to VAR back-testing, seems more appropriate for AI-based systems than for manual analysis. The best-fitting theoretical probability distribution may not necessarily provide the most accurate estimates for VaR or ES.

Recent advances in extreme risk quantification include applications of neural networks and LLMs. Evidence also suggests that a simple neural network incorporating long short-term memory can be an effective model for explaining conditional variance, comparable to traditional approaches. Such models can easily address commonly occurring, non-linear relationships (Buczynski & Chlebus, 2024). The evidence on LLM-generated VaR and ES forecasts produced via prompt interaction tested against standard and extended GARCH and EWMA also shows that LLMs may produce viable results (Pele et al., 2026).

The existing literature clearly demonstrates that skewed and leptokurtic distributions may outperform both the classical Gaussian parametric model and the historical simulation method. This paper focuses on applying the Laplace distribution (LD) and asymmetrical Laplace distribution (ALD, Kozubowski & Podgórski, 2000) for estimating parametric VaR. The in-depth literature review identifies a research gap: evidence of both symmetrical and asymmetrical Laplace distributions in the context of VaR estimation remains limited.

2. Methodology and data

The research procedure aims to achieve two primary objectives: (1) to back-test symmetrical and asymmetrical Laplace distributions and assess their ability to capture extreme events, and (2) to determine whether the distribution that best fits the entire empirical distribution is also the one that performs best in back-testing in the long term.

A long-term standard Kupiec test (Jędrusik et al., 2007; Kupiec, 1995) was performed for the 0.99 confidence level VaR to address extreme market risk, and additionally for the 0.975 and 0.95 to check lower extreme market risk levels. The distributions considered include the empirical distribution (E), normal distribution (N), Laplace distribution (L), and asymmetrical Laplace distribution (AL). In the standard Kupiec test, the LRUC statistic exceeding 3.84 and 6.63 for 0.05 and 0.01 significance levels, respectively, necessitates the rejection of the null hypothesis H_0: $\pi = \alpha$, where π is the observed violation rate and α is the significance level. Alternatively, the p-value for LRUC can be calculated, and H_0 is rejected whenever it is lower than α. The significance level applied for the Kupiec test was 0.01. The Kupiec test evaluates accuracy, meaning a model is rejected when the violation ratio is significantly higher or lower than the assumed significance level. These two cases differ fundamentally. When the null hypothesis H_0 is rejected and the violation ratio is lower than the significance level, the model is deemed conservative. Such models tend to overestimate risk, which is still preferable to underestimating. A violation ratio close to 1 classifies the model as neutral. Regarding volatility clustering, the Christoffersen test (1998) is often used alongside the Kupiec test as a conditional coverage measure, as it assesses not only the exceedance rate but also the independence of exceedances. However, given the very long sample period considered in the paper (over 25 years and 6,379 daily returns), which encompasses extended crisis periods such as 2008, a full-sample independence test would be difficult to interpret. The sample is affected by regime changes, volatility clustering, and structural breaks, which would violate the stationarity assumptions underlying the test. As a result, models could fail the Christoffersen test for reasons unrelated to their actual forecasting quality. Therefore, the Kupiec test was chosen as the back-testing tool and the unconditional coverage was considered.

The indices (representing respective capital markets) analysed include WIG30 (Warsaw), BVP (BOVESPA, Sao Paulo), CAC (CAC40, Paris), DAX (DAX40, Frankfurt), FTM (FTSE250, London), HSI (Hang Seng, Hong Kong), NKX (Nikkei225, Tokio), SHC (Shanghai), SPX (S&P500, New York) and TWSE (TAIEX, Taiwan). These are mostly classified as large cap indices (except SHC, FTM and TWSE), as well as being commonly perceived or classified as main indices for the markets considered. The indices were chosen to ensure that the long-term VaR test was carried out on a broad and heterogeneous set of equity markets. The set of indices includes major benchmarks from Europe, US, Latin America, and Asia, therefore covering both developed and emerging markets. This range is important because VaR models may behave differently across markets with different volatility patterns and different exposure to extreme events, whether global or local in nature. Altogether, this set allows for a more robust assessment of whether the VaR model using Laplace distribution or its asymmetrical counterpart performs consistently across different market environments in long-term.

The tests were performed using daily data obtained from stooq.pl for the period between 31 December 1998, and 30 June 2025 (6,379 daily simple rates of return; the detailed descriptive statistics are provided in Appendix 1, Table A1). Simple rates of return were applied, since these do not alter the substantive conclusions of the analysis, while also allowing for a more straightforward calculation procedure (Jorion, 2007; Miskolczi, 2017). The simple rates cannot be applied for time series models, e.g., geometric Brownian motion which rely on continuous compounding (Black & Scholes, 1973; Brigo et al., 2007; Kaczmarzyk, 2022). The safety level, often referred to as VaR in research, was used as the extreme risk measure. A 250-day window was applied. For each window, the parameters of the theoretical distributions were fitted and then used to calculate the safety level using the respective inverse function of a theoretical cumulative probability distribution function.

Kolmogorov–Smirnov statistics (K-S) and Akaike's Information Criterion (AIC) (Akaike, 1998; Vose, 2008) were used in parallel on each 250-day window to determine whether the best-fitting theoretical distribution (best fitting to the entire empirical distribution) also performs best in back-testing over the long term. The K-S test considers the maximum distance between the theoretical and empirical cumulative distribution functions (focusing on a single point, but scanning the whole distribution), while AIC evaluates the overall distribution fit. While the Anderson–Darling statistic explicitly emphasises tail behaviour, the Schwarz (BIC) and Hannan–Quinn (HQC) information criteria impose stricter penalties on model complexity (stronger than AIC). These harsh penalties may discourage the selection of additional parameters such as skewness, even when these parameters improve tail representation. AL would be then penalised more easily with BIC and HQC, making AIC preferable when sensitivity to distributional asymmetry is desired (Appendix 2, Table A2).

The financial data were obtained from stooq.pl. Both the parameter fitting and inverse function calculations were carried out using the SciPy 1.13.1 library.

Additionally, periods of significant market decline, interpreted as crisis periods, are highlighted in the figures. Figures 1–10 provided in Supplementary materials, Appendix A present each capital market and include (1) standardised index value (PV = 1), (2) AIC values, and (3) the results of back-testing (0.99 confidence level). The periods marked include the Internet bubble (A), the financial crisis (B1, B2), the Eurozone crisis (C), Brexit (D), COVID-19 (E) and the war in Ukraine (F). The visualisation of back-testing for 0.975 and 0.95 confidence levels is provided in Supplementary materials, Appendix B and C.

3. Results

Considering the extreme risk at the 0.99 confidence level (Table 1), neither E nor N passed the Kupiec test for any of the analysed indices. Both L and AL passed the test for WIG30, CAC, DAX, FTM, HSI, NKX, and SHC (7 out of 10 analysed indices). Additionally, AL passed the test for SPX and TWSE, increasing its coverage for 9 out of 10 indices. The LR_{UC} value for AL was lower than that for L for BVP, CAC, FTM, SHC, SPX, and TWSE. AL outperformed L, which may indicate skewness of the index (and the respective market). The Violation Ratio (VR) was less than or equal to 1 for both L and AL for WIG30 and BVP. L and AL can thus be considered conservative options, with closely matching VR values (0.894 and 0.894 respectively) for WIG30. For BVP, both VR values were significantly lower (0.627 and 0.643 respectively). A VR lower than 1 was calculated for DAX for AL.

Regarding the slightly lower extreme risk at the 0.975 confidence level (Table 2), neither E nor N passed the Kupiec test for CAC, DAX, FTM, NKX, SPX, and TWSE (6 out of 10 analysed indices). Additionally, N did not meet the critical LR_{UC}value for WIG30, HSI, and SHC. Either L or AL recorded the lowest LR_{UC} value for CAC, DAX, FTM, HSI, NKX, SHC, SPX, and TWSE (8 of the 10 indices considered), beating E and N as a VaR model. All these indices passed the critical criterion of the Kupiec test except SPX. The LR_{UC} value for AL was lower than that for L for CAC, DAX, FTM, HIS, SHC, SPX and TWSE. While the tent-shaped models did not pass the Kupiec test for WIG30 and BVP, they did offer VRs significantly lower than 1 (all lower than 0.8 for both indices), being a very conservative choice in the case of these indices. Among the indices that passed the Kupiec test, the only index with a VR lower than 1 was HSI.

Considering the 0.95 confidence level (Table 3), E and N passed the Kupiec test for all indices except DAX (9 out of 10). Both tent-shaped L and AL satisfied the back-testing criterion for BVP, CAC, FTM, HSI, NKX, SHC, SPX, and TWSE (8 out of 10 analysed indices). Additionally, AL alone met the test criterion for DAX. In the case of CAC, DAX, and FTM, either L or AL indicated the lowest LR_{UC}, beating E or N (with AL beating L for all 3 indices). AL was the best option for SPX and TWSE, and L the best for NKX. WIG30 did not pass the Kupiec test for the 0.95 confidence level VaR, despite having a VR lower than 1 (0.812 for L and 0.856 for AL), the lowest violation ratio among the indices considered. In terms of the Kupiec test, L and AL are too conservative for WIG30. A VR lower than 1 was observed for BVP and NKX for both L and AL (0.919 and 0.906 for BVP, respectively, and 0.919 and 0.959 for NKX). Additionally, AL proved to be a reliable conservative option for CAC and FTM.

In summary, the tent-shaped L or AL models passed the long-term Kupiec test for all confidence levels for CAC, FTM, HSI, NKX, and SHC (5 out of 10 indices). They offered LR_{UC} values lower than E or N for 0.99 and 0.975 confidence level VaR for CAC, DAX, FTM, HSI, NKX, SHC, SPX, and TWSE (8 out of 10 indices). They recorded the lowest LRUC for all confidence levels for CAC, DAX, and FTM. Given that the test was carried out on a broad and heterogeneous set of equity markets, L or AL seem to be reliable models for VaR. Generally, the higher the level of extreme risk, the more strongly this claim is supported. Given the shape of L and AL, and the back-testing results, extreme market risk for the analysed indices can be characterised as predominantly tent-shaped especially in term of higher confidence levels (0.99 and 0.975).

Regarding the back-testing results, the tent-shaped distributions work best for the highest level of market risk considered. The next question is whether the AIC and K-S criteria can correctly identify the best parametric model for the highest extreme risk measurement. According to AIC, L was most frequently the best-fitting model for all indices except WIG30 and BVP, where N was identified as offering the best fit. Although LR_{UC} indicated that AL often outperformed L, AIC did not confirm this, as AL was less frequently selected as the best fit. AIC confirmed the relationship between LR_{UC}, and model fit (meaning the distribution with the lower LR_{UC} tends to be more often best-fitting) for DAX, HSI, and NKX (3 out of 10 indices). These results suggest symmetrical market risk characteristics for those indices.

The AIC criterion favoured N over AL for WIG30, BVP, CAC, DAX, FTM, HSI, NKX, and SPX (8 out of 10 indices). Both N and L are symmetrical distributions. The AIC criterion evaluates how closely a probability distribution fits the overall data (based on the sum of log-likelihood values). Nonetheless, AIC results also support the tent-shaped nature of market risk. Combining the frequency with which L or AL were identified as the best-fitting models, they outperformed N for 9 out of 10 indices (all except BVP). Considering the combined frequency (the total percentage of observations with minimum AIC for L and AL, Table 1),

Table 1. Back-testing results with 0.99 confidence level for safety level (VaR) and moving AIC and K-S

Market / index	WIG30				BVP				CAC				DAX				FTM			
PD Type	(2) E	(3) N	(4) L	(5) AL	(2) E	(3) N	(4) L	(5) AL	(2) E	(3) N	(4) L	(5) AL	(2) E	(3) N	(4) L	(5) AL	(2) E	(3) N	(4) L	(5) AL
Observations	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379
Significance	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%
Observed	1.50%	1.60%	0.90%	0.90%	1.40%	1.60%	0.60%	0.60%	1.60%	2.30%	1.20%	1.00%	1.60%	2.20%	1.10%	0.90%	1.70%	2.60%	1.30%	1.10%
Expected violations	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79
Actual violations	98	101	57	57	88	103	40	41	105	145	75	65	103	139	68	58	108	168	85	71
Violation Ratio	1.536	1.583	0.894	0.894	1.38	1.615	0.627	0.643	1.646	2.273	1.176	1.019	1.615	2.179	1.066	0.909	1.693	2.634	1.332	1.113
L0	−514.43	−528.22	−326.03	−326.03	−468.48	−537.41	−247.92	−252.51	−546.60	−730.40	−408.75	−362.79	−537.41	−702.83	−376.58	−330.63	−560.38	−836.09	−454.70	−390.37
L1	−506.47	−518.91	−325.65	−325.65	−464.33	−527.15	−242.75	−247.80	−535.35	−692.02	−407.80	−362.78	−527.15	−669.33	−376.44	−330.35	−547.57	−776.75	−451.47	−389.97
LRUC	15.922	18.624	0.757	0.757	8.299	20.525	10.332	9.416	22.506	76.759	1.884	0.023	20.525	67.007	0.275	0.548	25.622	118.681	6.451	0.794
p-value	0	0	0.384	0.384	0.004	0	0.001	0.002	0	0	0.17	0.879	0	0	0.6	0.459	0	0	0.011	0.373
Violation Ratio (VR) <= 1	☒	☒	☑	☑	☒	☒	☑	☑	☒	☒	☒	☒	☒	☒	☒	☑	☒	☒	☒	☒
p-value > 0.01	☒	☒	☑	☑	☒	☒	☒	☒	☒	☒	☑	☑	☒	☒	☑	☑	☒	☒	☑	☑
LRUC(AL) < LRUC(L)				☒				☑				☑				☒				☑
Obser. with min (AIC)		2,822	2,479	1,078		3,509	2,277	593		2,022	3,097	1,260		1,743	3,614	1,022		1,931	2,879	1,569
%		44.24	38.86	16.90		55.01	35.70	9.30		31.70	48.55	19.75		27.32	56.65	16.02		30.27	45.13	24.60
AIC conf. LRUC				☒				☒				☒				☑				☒
Observ. with min (K-S)		2,720	1,431	2,228		2,832	1,343	2,204		1,927	1,472	2,980		1,229	1,433	3,717		2,019	2,051	2,309
%		42.64	22.43	34.93		44.40	21.05	34.55		30.21	23.08	46.72		19.27	22.46	58.27		31.65	32.15	36.20
K-S conf. LRUC				☒				☒				☑				☒				☑

Market	HSI				NKX				SHC				SPX				TWSE			
PD Type	(2) E	(3) N	(4) L	(5) AL	(2) E	(3) N	(4) L	(5) AL	(2) E	(3) N	(4) L	(5) AL	(2) E	(3) N	(4) L	(5) AL	(2) E	(3) N	(4) L	(5) AL
Observations	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379
Significance	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%
Observed	1.60%	2.00%	1.10%	1.10%	1.60%	2.20%	1.10%	1.20%	1.40%	2.10%	1.30%	1.30%	1.70%	2.60%	1.60%	1.30%	1.50%	2.20%	1.40%	1.30%
Expected violations	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79	63.79
Actual violations	104	129	67	73	104	141	70	78	87	134	81	80	110	164	99	85	95	142	89	84
Violation Ratio	1.63	2.022	1.05	1.144	1.63	2.21	1.097	1.223	1.364	2.101	1.27	1.254	1.724	2.571	1.552	1.332	1.489	2.226	1.395	1.317
L0	−542.00	−656.88	−371.98	−399.56	−542.00	−712.02	−385.77	−422.53	−463.89	−679.86	−436.32	−431.72	−569.57	−817.71	−519.03	−454.70	−500.65	−716.62	−473.08	−450.10
L1	−531.25	−630.91	−371.90	−398.91	−531.25	−676.92	−385.47	−421.04	−460.06	−650.22	−434.16	−429.80	−555.68	−762.26	−510.63	−451.47	−493.94	−680.71	−468.60	−447.16
LRUC	21.506	51.943	0.161	1.283	21.506	70.2	0.592	2.986	7.66	59.285	4.321	3.85	27.793	110.899	16.802	6.451	13.408	71.818	8.962	5.882
p-value	0	0	0.689	0.257	0	0	0.442	0.084	0.006	0	0.038	0.05	0	0	0	0.011	0	0	0.003	0.015
Violation Ratio (VR) <= 1	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒
p-value > 0.01	☒	☒	☑	☑	☒	☒	☑	☑	☒	☒	☑	☑	☒	☒	☒	☑	☒	☒	☒	☑
LRUC(AL) < LRUC(L)				☒				☒				☑				☑				☑
Obser. with min (AIC)		1,102	4,618	659		1,277	4,635	467		3	5,189	1,187		1,378	4,481	520		275	5,474	630
%		17.28	72.39	10.33		20.02	72.66	7.32		0.05	81.35	18.61		21.60	70.25	8.15		4.31	85.81	9.88
AIC conf. LRUC				☑				☑				☒				☒				☒
Observ. with min (K-S)		1,059	1,941	3,379		904	1,909	3,566		39	2,648	3,692		966	1,793	3,620		247	2,573	3,559
%		16.60	30.43	52.97		14.17	29.93	55.90		0.61	41.51	57.88		15.14	28.11	56.75		3.87	40.34	55.79
K-S conf. LRUC				☒				☒				☑				☑				☑

Source: own elaboration.

Table 2. Back-testing results with 0.975 confidence level for safety level (VaR)

Market / index	WIG30				BVP				CAC				DAX				FTM			
PD Type	(2) E	(3) N	(4) L	(5) AL	(2) E	(3) N	(4) L	(5) AL	(2) E	(3) N	(4) L	(5) AL	(2) E	(3) N	(4) L	(5) AL	(2) E	(3) N	(4) L	(5) AL
Observations	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379
Significance	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%
Observed	3.00%	3.00%	1.80%	2.00%	2.90%	3.00%	1.80%	1.90%	3.30%	3.70%	2.70%	2.50%	3.40%	3.80%	2.70%	2.60%	3.30%	3.70%	3.00%	2.60%
Expected violations	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475
Actual violations	191.000	193.000	115.000	125.000	185.000	192.000	115.000	123.000	209.000	235.000	170.000	161.000	216.000	244.000	174.000	166.000	213.000	238.000	192.000	166.000
Violation Ratio	1.198	1.210	0.721	0.784	1.160	1.204	0.721	0.771	1.311	1.474	1.066	1.010	1.354	1.530	1.091	1.041	1.336	1.492	1.204	1.041
L0	–861.243	–868.570	–582.812	–619.448	–839.261	–864.906	–582.812	–612.120	–927.187	–1022.439	–784.308	–751.336	–952.832	–1055.411	–798.962	–769.654	–941.841	–1033.430	–864.906	–769.654
L1	–858.234	–865.179	–575.778	–615.324	–837.267	–861.709	–575.778	–607.482	–919.991	–1006.395	–783.959	–751.328	–943.566	–1035.591	–798.303	–769.518	–933.492	–1016.166	–861.709	–769.518
LRUC	6.018	6.781	14.068	8.247	3.988	6.394	14.068	9.277	14.392	32.089	0.698	0.015	18.530	39.641	1.318	0.270	16.698	34.528	6.394	0.270
p-value	0.014	0.009	0.000	0.004	0.046	0.011	0.000	0.002	0.000	0.000	0.404	0.903	0.000	0.000	0.251	0.603	0.000	0.000	0.011	0.603
Violation Ratio (VR) <= 1	☒	☒	☑	☑	☒	☒	☑	☑	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒
p-value > 0.01	☑	☒	☒	☒	☑	☑	☒	☒	☒	☒	☑	☑	☒	☒	☑	☑	☒	☒	☑	☑
Market	**HSI**				**NKX**				**SHC**				**SPX**				**TWSE**			
PD Type	(2) E	(3) N	(4) L	(5) AL	(2) E	(3) N	(4) L	(5) AL	(2) E	(3) N	(4) L	(5) AL	(2) E	(3) N	(4) L	(5) AL	(2) E	(3) N	(4) L	(5) AL
Observations	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379
Significance	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%	2.50%
Observed	3.00%	3.20%	2.40%	2.40%	3.30%	3.60%	2.70%	2.70%	2.90%	3.30%	2.80%	2.60%	3.40%	3.90%	3.30%	3.10%	3.00%	3.60%	2.80%	2.80%
Expected violations	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475	159,475
Actual violations	192.000	204.000	151.000	156.000	210.000	232.000	171.000	174.000	188.000	208.000	179.000	165.000	215.000	251.000	210.000	195.000	193.000	232.000	181.000	177.000
Violation Ratio	1.204	1.279	0.947	0.978	1.317	1.455	1.072	1.091	1.179	1.304	1.122	1.035	1.348	1.574	1.317	1.223	1.210	1.455	1.135	1.110
L0	–864.906	–908.869	–714.700	–733.018	–930.850	–1011.449	–787.971	–798.962	–850.252	–923.523	–817.280	–765.990	–949.168	–1081.056	–930.850	–875.897	–868.570	–1011.449	–824.607	–809.953
L1	–861.709	–903.003	–714.465	–732.979	–923.373	–996.584	–787.554	–798.303	–847.775	–916.603	–816.100	–765.893	–940.213	–1058.060	–923.373	–872.103	–865.179	–996.584	–823.178	–808.999
LRUC	6.394	11.733	0.470	0.078	14.954	29.730	0.835	1.318	4.954	13.840	2.360	0.194	17.910	45.994	14.954	7.587	6.781	29.730	2.857	1.908
p-value	0.011	0.001	0.493	0.780	0.000	0.000	0.361	0.251	0.026	0.000	0.124	0.659	0.000	0.000	0.000	0.006	0.009	0.000	0.091	0.167
Violation Ratio <= 1	☒	☒	☑	☑	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒
p-value > 0.01	☑	☒	☑	☑	☒	☒	☑	☑	☑	☒	☑	☑	☒	☒	☒	☒	☒	☒	☑	☑

Source: own elaboration.

Table 3. Back-testing results with 0.95 confidence level for safety level (VaR)

Market / index	**WIG30**				**BVP**				**CAC**				**DAX**				**FTM**			
PD Type	(2) E	(3) N	(4) L	(5) AL	(2) E	(3) N	(4) L	(5) AL	(2) E	(3) N	(4) L	(5) AL	(2) E	(3) N	(4) L	(5) AL	(2) E	(3) N	(4) L	(5) AL
Observations	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379
Significance	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%
Observed	5.30%	4.60%	4.10%	4.30%	5.40%	5.30%	4.60%	4.50%	5.70%	5.50%	5.20%	4.90%	6.00%	6.20%	5.90%	5.40%	5.70%	5.70%	5.60%	5.00%
Expected violations	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95
Actual violations	339.000	291.000	259.000	273.000	346.000	339.000	293.000	289.000	364.000	354.000	333.000	314.000	380.000	398.000	374.000	344.000	364.000	364.000	355.000	316.000
Violation Ratio	1.063	0.912	0.812	0.856	1.085	1.063	0.919	0.906	1.141	1.110	1.044	0.984	1.191	1.248	1.173	1.079	1.141	1.141	1.113	0.991
L0	-1325.365	-1184.032	-1089.810	-1131.032	-1345.976	-1325.365	-1189.921	-1178.143	-1398.976	-1369.531	-1307.698	-1251.754	-1446.087	-1499.087	-1428.420	-1340.087	-1398.976	-1398.976	-1372.476	-1257.643
L1	-1324.714	-1182.705	-1083.489	-1127.377	-1344.799	-1324.714	-1188.780	-1176.617	-1395.766	-1367.571	-1307.377	-1251.713	-1440.276	-1489.495	-1423.670	-1339.076	-1395.766	-1395.766	-1370.404	-1257.628
LRUC	1.301	2.653	12.640	7.310	2.353	1.301	2.282	3.053	6.419	3.921	0.643	0.081	11.621	19.184	9.499	2.022	6.419	6.419	4.144	0.029
p-value	0.254	0.103	0.000	0.007	0.125	0.254	0.131	0.081	0.011	0.048	0.423	0.776	0.001	0.000	0.002	0.155	0.011	0.011	0.042	0.865
Violation Ratio (VR) <= 1	☒	☑	☑	☑	☒	☒	☑	☑	☒	☒	☒	☑	☒	☒	☒	☒	☒	☒	☒	☑
p-value > 0.01	☑	☑	☒	☒	☑	☑	☑	☑	☑	☑	☑	☑	☒	☒	☒	☑	☑	☑	☑	☑
Market	HSI				NKX				SHC				SPX				TWSE			
PD Type	(2) E	(3) N	(4) L	(5) AL	(2) E	(3) N	(4) L	(5) AL	(2) E	(3) N	(4) L	(5) AL	(2) E	(3) N	(4) L	(5) AL	(2) E	(3) N	(4) L	(5) AL
Observations	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379	6,379
Significance	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%
Observed	5.20%	5.00%	4.60%	4.80%	5.50%	5.30%	5.10%	5.40%	5.30%	5.00%	5.20%	5.10%	5.50%	5.70%	5.60%	5.40%	5.60%	5.40%	5.30%	5.40%
Expected violations	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95	318,95
Actual violations	333.000	317.000	293.000	306.000	348.000	338.000	328.000	342.000	341.000	318.000	333.000	326.000	349.000	363.000	359.000	345.000	356.000	342.000	341.000	343.000
Violation Ratio	1.044	0.994	0.919	0.959	1.091	1.060	1.028	1.072	1.069	0.997	1.044	1.022	1.094	1.138	1.126	1.082	1.116	1.072	1.069	1.075
L0	-1307.698	-1260.587	-1189.921	-1228.198	-1351.865	-1322.420	-1292.976	-1334.198	-1331.254	-1263.532	-1307.698	-1287.087	-1354.809	-1396.031	-1384.254	-1343.031	-1375.420	-1334.198	-1331.254	-1337.142
L1	-1307.377	-1260.581	-1188.780	-1227.918	-1350.510	-1321.832	-1292.842	-1333.341	-1330.468	-1263.530	-1307.377	-1287.006	-1353.361	-1392.960	-1381.705	-1341.939	-1373.234	-1333.341	-1330.468	-1336.210
LRUC	0.643	0.013	2.282	0.561	2.708	1.176	0.268	1.715	1.571	0.003	0.643	0.163	2.895	6.142	5.096	2.184	4.373	1.715	1.571	1.865
p-value	0.423	0.911	0.131	0.454	0.100	0.278	0.605	0.190	0.210	0.956	0.423	0.686	0.089	0.013	0.024	0.139	0.037	0.190	0.210	0.172
Violation Ratio <= 1	☒	☑	☑	☑	☒	☒	☒	☒	☒	☑	☒	☒	☒	☒	☒	☒	☒	☒	☒	☒
p-value > 0.01	☑	☑	☑	☑	☑	☑	☑	☑	☑	☑	☑	☑	☑	☑	☑	☑	☑	☑	☑	☑

Source: own elaboration.

WIG30 exhibits the second-lowest value, whereas TWSE and SHC record the highest values (exceeding 90%). Brazilian and European markets are associated with the five lowest outcomes, while Asian and American markets account for the five highest. Nevertheless, the L and AL approaches perform best for WIG30 and BVP when applied for extreme VaR calculations. According to AIC, these models achieve the lowest combined frequency.

Graphical assessments reveal that AIC might become significantly lower for both L and AL at the onset of a crisis, thus favouring these distributions. This appears to be a recurring phenomenon (Table 4; Supplementary materials, Appendix A: Figures 1–10). Considering AIC, the most of analysed indices gave a clear response to COVID-19 (7 out of 10). According to the AIC criterion, most of the analysed indices reacted significantly to the COVID-19 pandemic (7 out of 10). However, the Polish index was the only one to exhibit an evident response to the war in Ukraine; it was also the only index showing a clear reaction to the Eurozone crisis. A similar pattern was observed for Brexit, where an evident response occurred only in the United Kingdom. Furthermore, based on AIC, most analysed indices displayed no reaction or only a slight reaction during the dot-com bubble and the global financial crisis.

Table 4. Differences in AIC values for the considered distributions during crisis periods

Crisis	WIG30	BVP	CAC	DAX	FTM	HSI	NKX	SHC	SPX	TWSE
A: Internet bubble	☒	☒	☒	☒	☑	☒	☒	☑	☒	☒
B1: Financial crisis	☒	☒	☒	☒	☒	☑	☑	☑,☒	☑	☑
B2: Financial crisis	☒	☑.☑	☑	☑	☒	☑	☑.☑	☒	☑	☒
C: Eurozone crisis	☑.☑	☒	☒	☑	☒	☑	☑,☒	☒	☑	☑
D: Brexit	☒	☒	☒	☒	☑.☑	☒	☑,☒	☒	☒	☑,☒
E: COVID-19	☑.☑	☑.☑	☑.☑	☑.☑	☑.☑	☑	☑.☑	☑,☒	☑.☑	☑.☑
F: War in Ukraine	☑.☑	☒	☑	☑	☒	☑	☒	☑	☒	☒
☑.☑ evident, ☑ slightly evident, ☒ no reaction, or AIC started receding earlier, ☑,☒ hard to decide										

Source: own elaboration.

The Kolmogorov-Smirnov test is easier to implement and different, because it focuses only on the maximum difference between cumulative distribution functions at a specific point scanning the entire distribution. The K-S test confirmed the relationship between LR_{UC} and model fit (more frequently chosen probability distributions correspond to lower LR_{UC} values) for BVP, CAC, FTM, SHC, SPX, and TWSE (6 out of 10 markets), which had lower LR_{UC} values for AL, indicating asymmetrical characteristics for the highest level of extreme market risk considered.

According to K-S statistics, L or AL consistently outperformed the normal distribution for all 10 analysed indices. In terms of the combined frequency (the total percentage of observations with minimum K-S for L and AL, Table 1), WIG30 and BVP registered the lowest values, whereas TWSE and SHC recorded the highest values (exceeding 90%). As observed again, Brazilian and European markets are associated with the five lowest outcomes, while Asian and American markets account for the five highest. Again, the L and AL approaches performing best for WIG30 and BVP when applied for extreme VaR calculation have the lowest combined frequency according to K-S.

The VaR back-testing results reveal clear differences across the analysed markets and across confidence levels. Although both the Laplace and asymmetric Laplace distributions generally outperformed the empirical and normal distributions, their relative effectiveness was not uniform across indices or risk levels. In the case of WIG30, both Laplace-based models passed the Kupiec test at the 0.99 confidence level with a violation ratio slightly below 1, suggesting a relatively balanced and conservative fit; however, for lower confidence levels (0.975 and 0.95), both models became overly conservative and did not pass the test despite maintaining violation ratios below 1. A similar pattern was partially observed for DAX, where symmetric behaviour was evident, although the asymmetric Laplace distribution additionally met the Kupiec criterion at the 0.95 level.

In several other markets, the asymmetric Laplace distribution achieved lower LR_{UC} values than the symmetric Laplace distribution, indicating that market risk was not only fat-tailed but also asymmetric in terms of extreme downside risk. This was particularly visible for BVP, CAC, FTM, SHC, SPX and TWSE, where the asymmetric specification more accurately captured tail risk, especially at the 0.99 and 0.975 levels, and often remained competitive at 0.95. By contrast, in markets such as DAX, HSI and NKX, the evidence more often pointed to the symmetric Laplace distribution, suggesting a more symmetric tent-shaped risk profile, although this distinction became less pronounced at lower confidence levels, where both specifications frequently produced similar and acceptable back-testing outcomes.

Overall, the inclusion of multiple confidence levels confirms that while excess kurtosis is a common feature across all markets, the degree of asymmetry and the relative advantage of the asymmetric Laplace distribution are more pronounced at higher confidence levels, whereas at lower levels the differences between symmetric and asymmetric specifications tend to diminish, and model conservatism becomes more relevant than distributional shape.

These cross-market differences were also reflected in the goodness-of-fit results. According to AIC, the symmetric Laplace distribution was more frequently selected as the best-fitting model, whereas the K-S criterion more often favoured the asymmetric Laplace distribution, especially in markets where the VaR back-testing results indicated stronger asymmetry. At the same time, the dynamic behaviour of AIC during crisis episodes showed that both Laplace-based distributions tended to improve their relative fit when market stress intensified. This pattern was particularly visible during the COVID-19 period, which generated the clearest and most widespread response across the analysed indices, while other crises were more market-specific, such as Brexit in the UK market or the war in Ukraine, which was most clearly reflected in WIG30.

Overall, the results suggest that although excess kurtosis is a common feature across all analysed capital markets represented by respective indices, the degree of skewness, the preferred distributional fit, and the sensitivity to crisis episodes all differ substantially across markets. The back-testing results and distribution fitting consistently support the interpretation that market risk is predominantly tent-shaped in most of the capital markets analysed, especially for the higher levels of extreme market risk.

4. Discussion

The results obtained for the 0.99 confidence level are consistent with previous research on applying L for extreme market risk measurement (Ullah et al., 2022). Selecting L or AL is generally preferable to relying on E or N. This study further confirms that relying on E is consistently more effective than using N for all 10 analysed indices, a result commonly observed in existing research. Whenever L outperforms E or N, it indicates a market/index characterised by fat tails (leptokurtosis). However, AL (Kozubowski & Podgórski, 2000) can be considered a more appropriate choice than L. Specifically, L or AL outperformed E or N for 9 out of 10 indices, while AL outperformed L for 6 indices (5 if excluding BVP, where N was better than AL). This clearly indicates that main indices representing respective markets are often not only leptokurtic but also exhibit skewness, fully aligning with previous research findings on extreme risk measurement using non-Laplace distributions (e.g., Altun et al., 2018; Bormetti et al., 2007; Braione & Scholtes, 2016; C. F. Lee & Su, 2012). Extending the analysis to lower confidence levels (0.975 and 0.95) provides additional insight into the robustness of these findings. At the 0.975 level, the superiority of the Laplace-based models over the empirical and normal distributions remains evident, although their performance becomes less uniform across markets and, in some cases, more conservative, as reflected in violation ratios below 1. At the 0.95 level, the differences between models diminish further, with the empirical and normal distributions more frequently passing the Kupiec test, while L and AL still often provide competitive or superior LR_{UC} values for several indices. These results suggest that the relative advantage of tent-shaped distributions is strongest in modelling extreme market risk, whereas at lower confidence levels the choice of distribution becomes less critical and model conservatism plays a more prominent role.

Applying moving probability distribution fitting is not a common practice in back-testing procedures. Although using criteria such as AIC or K-S in back-testing does not conclusively determine the best parametric extreme risk model, it does clearly indicate the fat-tailed nature of the analysed indices and consequently, of the capital markets they represent. According to the AIC, L was identified as the best-fitting model for 8 out of 10 indices, while according to the K-S criterion, AL was best-fitting for 8 out of 10 markets for the highest extreme market risk level considered. The AIC criterion coincided with markets where LR_{UC} values were lower for L, whereas the K-S criterion coincided with markets where LR_{UC} values were lower for AL. Additionally, graphical analysis revealed instances (aligned with crises) where the AIC of L or AL deviated significantly from the AIC of N.

Conclusions

Capital markets exhibit extreme events that the normal distribution struggles to capture. While the simplicity of the normal distribution is an argument in its favour, estimating extreme risk ad hoc could prove problematic. Much of the research so far confirms that relying solely on the empirical distribution or normal distribution is an unreliable solution. However, the empirical distribution is usually a better choice than its normal counterpart.

This research procedure confirmed the superiority of tent-shaped Laplace and asymmetrical Laplace distributions over empirical and normal distributions in extreme market risk measurement for 0.99 confidence level. These can be considered reliable parametric models. Across the ten markets studied, considering the 0.99 confidence level, the Laplace distribution and its asymmetrical counterpart generally outperformed historical simulation and normal distribution, with the asymmetrical Laplace distribution particularly outperforming the Laplace distribution in capital markets that are likely to be skewed. This was also confirmed for the 0.975 confidence level VaR. However, the superiority of tent-shaped distributions was not confirmed for VaR calculated with a 0.95 confidence level.

In the case of the Polish capital market, the Laplace and asymmetrical Laplace distributions not only pass the back-testing procedure for the 0.99 confidence level, but with a violation ratio slightly lower than 1, they could also be classified as convenient conservative/neutral models for extreme risk measurement. For lower confidence levels, while violation ratios were lower than 1, the long-term Kupiec test failed. The tent-shaped distributions occurred to be too conservative models for lower extreme risk levels.

Moving goodness-of-fit tests were not consistent between AIC and K-S: AIC identified the best choice mostly when the Kupiec test favoured Laplace, whereas K-S did so mostly when it favoured the asymmetrical Laplace distribution. Although using criteria such as AIC or K-S in back-testing does not conclusively determine the best risk model, it does clearly indicate the fat-tailed nature of the analysed capital markets.

The main limitation of this study lies in the long-term character of the back-testing procedure, which, although useful for evaluating model performance across multiple market regimes, may also mask shorter-term changes in distributional properties, structural breaks, and regime-specific dynamics. For this reason, the study focuses on unconditional coverage assessed with the Kupiec test. In a more detailed setting, especially with shorter subperiods, it would be possible to account for volatility clustering. In particular, the results obtained for the Polish capital market suggest that both the Laplace and the asymmetric Laplace distributions perform well in modelling very high extreme risk (0.99 confidence level for VaR) serving as conservative models, but these findings should be interpreted with caution. Another limitation is that the study focuses primarily on major stock indices; therefore, future research should extend the analysis to all relevant indices and/or major individual stocks in order to assess the broader applicability of Laplace-based VaR modelling in the capital market. More detailed tests involving individual stocks should also be conducted in those markets where the Laplace and asymmetric Laplace distributions proved to be a good choice for all the confidence levels considered.

Appendix 1

Table A1. Descriptive statistics for daily returns obtained from stooq.pl for the period between 31 December 1998, and 30 June 2025

Market / index	Count	Mean (%)	Standard deviation (%)	Min (%)	1%	5%	95%	99%	Max (%)
WIG	6,379	0.035	1.263	–12.652	–3.419	–1.908	2.045	3.282	7.716
WIG30	6,379	0.022	1.449	–13.090	–3.848	–2.252	2.331	3.913	8.421
^BVP	6,379	0.049	1.704	–14.780	–4.300	–2.667	2.581	4.373	14.658
^CAC	6,379	0.015	1.395	–12.277	–3.968	–2.188	2.072	3.611	11.176
^DAX	6,379	0.031	1.433	–12.239	–4.155	–2.258	2.196	3.710	11.402
^FTM	6,379	0.025	1.075	–9.353	–3.044	–1.668	1.560	3.085	8.371
^HSI	6,379	0.016	1.487	–13.223	–4.051	–2.271	2.250	3.937	14.348
^NKX	6,379	0.022	1.444	–12.396	–3.939	–2.233	2.151	3.604	14.150
^SHC	6,379	0.025	1.449	–8.841	–4.500	–2.248	2.237	3.952	9.857
^SPX	6,379	0.031	1.227	–11.984	–3.432	–1.860	1.746	3.416	11.580
^TWSE	6,379	0.023	1.302	–9.700	–3.854	–2.116	1.984	3.700	9.247

Source: own elaboration.

Appendix 2

Table 6. Penalties in information criteria, the dependency between size (*n*) of the window and the number of theoretical distribution parameters (*k*)

AIC							
k / *n*	100	250	400	550	700	850	1000
2	4.12	4.05	4.03	4.02	4.02	4.01	4.01
3	6.25	6.10	6.06	6.04	6.03	6.03	6.02
4	8.42	8.16	8.10	8.07	8.06	8.05	8.04
5	10.64	10.25	10.15	10.11	10.09	10.07	10.06
BIC							
k / *n*	100	250	400	550	700	850	1000
2	9.21	11.04	11.98	12.62	13.10	13.49	13.82
3	1.82	16.56	17.97	18.93	19.65	20.24	20.72
4	18.42	22.09	23.97	25.24	26.20	26.98	27.63
5	23.03	27.61	29.96	31.55	32.76	33.73	34.54
HQC							
k / *n*	100	250	400	550	700	850	1000
2	6.11	6.83	7.16	7.37	7.52	7.64	7.73
3	9.16	10.25	10.74	11.05	11.28	11.45	11.60
4	12.22	13.67	14.32	14.74	15.04	15.27	15.46
5	15.27	17.09	17.90	18.42	18.80	19.09	19.33

Source: own elaboration.

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